Expert elicitation of autocorrelated time series with application to e3 (energy-environment-economic) forecasting models

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ABSTRACT

Explicitly representing uncertainty is recognised as a fundamental requirement of any long-term forecast. We propose and illustrate an expert elicitation protocol for constructing long-term probabilistic projections. Each projection represents a possible realization of a time series with autocorrelation properties, and thus a plausible future evolution of a quantity of interest. We illustrate the approach using two quantities – GDP growth rates and coal prices – that were elicited as part of a project producing baseline forecasts of greenhouse gas emissions in South Africa to 2050. The elicited projections can be used as inputs to deterministic structural models of the energy, economic, and environmental sectors (e3 or energy-environment-economic models), to generate similar probabilistic projections for any desired outputs of the e3 model. An R package for the generation and visualization of these probabilistic projections is provided.

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1. Introduction

Forecasts of greenhouse gas (GHG) emissions lie at the heart of many debates about the potential effects of climate change, the necessity for action, and the relative merits of different response strategies (Aman et al., 2011; Azar, Lindgren & Andersson, 2003; Pietzcker et al., 2014; O’Neill et al., 2010). One approach is to generate these forecasts as the outputs of e3 (energy-environment-economic) models, large-scale structural models that simultaneously capture relationships within and between the energy, economic, and environmental sectors. Using e3 models to compute GHG emissions allows one to forecast emissions indirectly, by passing forecasts of various drivers of emissions – such as population growth, economic growth, and commodity prices – to the e3 model. Forecasts of emissions are then obtained as a deterministic output of the e3 model. If probabilistic forecasts are available for the key drivers then this modelling approach becomes a Monte Carlo simulation that produces probabilistic forecasts of GHG emissions as an output. Here, the Monte Carlo approach effectively acts as a mechanism for the propagation of input uncertainty, and is particularly appropriate for non-linear systems such as those encountered in e3 modelling, for which simpler linear methods of uncertainty propagation are inappropriate. Parameter uncertainty in the underlying deterministic e3 model can also be considered using the Monte Carlo approach.

This approach of course still requires forecasts of any inputs to the e3 model, which may itself be a challenging task. Forecasts for some inputs e.g. population growth, may exist in secondary sources (Raftery et al., 2012), while other forecasts e.g. learning rates for some technologies, may be obtained by further modelling (Nemet, 2006; Masini and Franckl, 2003). In this paper we describe how forecasts can be obtained using expert elicitation, a structured interview procedure used to gather subjective judgements from domain experts (Krueger et al., 2012; Anadon et al., 2013). The forecasts that we seek are probabilistic in nature – that is, they explicitly include assessments of statistical uncertainty around a modal or most-likely forecast. These will, in turn, allow us to assess uncertainty in the forecasted GHG emissions.

Eliciting subjective probability judgments from experts is a well-established field, and a number of detailed protocols exist that encode a generally accepted best practice (e.g. O’Hagan et al., 2006).
The forecasts that we seek are somewhat different in that they are projections—a sequence of time-ordered observations that describe how an input variable, coal prices for example, might evolve over a long period of time, as well as an assessment of the uncertainty around these projections. This requires substantially more effort than the elicitation of one or a small number of probability distributions, which may be infeasible. Our approach is pragmatic, replacing the assessment of probabilistic projections with the assessment of probability distributions at selected time points, which can be conducted using existing protocols, followed by an interpolation between these time points using information also elicited from the expert. Together, the elicited probability distributions and interpolation information can be used to simulate possible projections for each input variable, and in turn these can be passed to the e3 model, which generates possible output projections for GHG emissions. We illustrate the approach using two quantities—GDP growth rates and coal prices—that were elicited as part of a project producing baseline forecasts of GHG emissions in South Africa to 2050. Although the context of our application is e3 modelling in South Africa, the elicitation methodology we use is broadly applicable to any situation in which time series with autocorrelation properties are to be assessed. We provide software, in the form of an R package, developed for the purpose of guiding the generation and visualization of these probabilistic projections.

Long-term forecasting is a controversial topic. It is often highly complex and subject to enormous uncertainties. Its failings have been well documented, as have motivations for the continued use of long-term forecasting (Craig et al., 2002). Nevertheless, in determining responses to climate change in the public sphere a quantitative forecasting approach remains popular. The current paper is an attempt to remain within the quantitative forecasting paradigm, but with the caveat that our outputs should be interpreted with the caution that must accompany all long-term forecasts.

The remainder of the paper is structured as follows. The next section gives a brief overview of using expert elicitation to assess uncertain quantities. We then describe the protocol that we used, including the elicitation of probability distributions as well as temporal information allowing us to interpolate between the assessed probability distributions. In Section 4 we describe the e3 model we use to compute GHG emissions. Sections 5 and 6 describe our two case studies: the elicitation of GDP growth rates and coal prices, Section 7 provides a general discussion of these and Section 8 concludes the paper. Details of algorithms used to generate projections and of the R package projection are given in Appendix A and B respectively.

2. Expert elicitation

Many practical problems require identifying the best available knowledge about certain quantities of interest. Our application, for example, seeks the best available estimates of how drivers of GHG emissions, such as GDP growth, population size, etc., are likely to change over time. Where past data provides only limited information about the future, a common approach is to seek the subjective judgment of experts who are most familiar with the specific area of interest. The processes whereby these expressions are obtained are referred to as “expert assessment” or “expert elicitation” procedures (e.g. O’Hagan et al., 2006; Garthwaite et al., 2005). Expert elicitation has been extensively used for energy and environmental applications (e.g. Baker et al., 2009; Anadon et al., 2013; see also the review by Krueger et al., 2012).

In order to be useful an expert’s knowledge needs to be encoded in a particular form—in our and many other cases this means in the form of a probability distribution. Unfortunately intuitive or unassisted assessments of probability are subject to systematic biases which, given certain conditions, can lead to substantial judgmental errors (e.g. Tversky and Kahneman, 1974; Gilovich et al., 2002). Thus, detailed protocols for eliciting probability distributions in such a way that these biases are avoided or at least minimized are well established (e.g. von Winterfeldt and Edwards, 1993; O’Hagan et al., 2006; Garthwaite et al., 2005). Various kinds of probability distributions can be elicited: marginal (outcomes involving a single variable), joint (outcomes involving two or more correlated variables e.g. Al-Awadhi and Garthwaite, 1998; Bedford and Cooke, 2001), and conditional (outcomes involving a single variable, in which other variables are held fixed at some values e.g. Kadane et al., 1980).

Implementing elicitation protocols requires substantial time and effort from the expert, usually under the guidance of a “facilitator” trained in elicitation methods. The aim of the elicitation is to accurately represent the expert’s judgment, including the degree to which they are uncertain about that judgment. An elicitation can therefore still be “good” even if the resulting judgment turns out to be badly wrong (Garthwaite et al., 2005). The quality of experts’ knowledge is not directly at issue, although of course the reason why they are consulted as experts in the first place is that their judgments are expected to be, within reason, sound. This means that the selection of experts is critical to the success of any process that makes use of the elicited judgments. Experts should be selected on the basis of technical knowledge and experience, but also so as to achieve a balanced set of views (Knol et al., 2010; RSC, 2004), bearing in mind that experts may also biased towards a particular viewpoint (Garthwaite et al., 2005). Formal selection procedures based on multi-stage nominations are described in RSC (2004) and applied in Knol et al. (2009). No precise guideline exists on the number of experts to use (Knol et al., 2010). A panel of elicitation practitioners recommend between 6 and 12 (Cooke and Probst, 2006), but Krueger et al. (2012) review a number of environmental applications using fewer (or more) experts. Practically, the number of experts used is likely to be largely a function of time and resource constraints, but will also vary with the stability of the underlying system, the degree to which strongly opposing views are held, and the need for public acceptance of the result (Knol et al., 2009).

3. Constructing long-term longitudinal forecasts using expert elicitation

Our goal is to produce, for any input variable of interest, a probabilistic projection, from which many possible input projections can be simulated and, in our case, passed to an e3 model. Directly assessing these projections is difficult or impossible. Long-term forecasts simply require too many assessments: in our case, five-yearly over the period 2015 to 2050. Even under the restrictive assumption of independence, this requires the assessment of eight probability distributions per input variable, a considerable but not impossible task. If independence is relaxed at least eight conditional distributions must be elicited, which will in most cases be infeasible.

Our approach is to elicit probability distributions at a subset of time points, and then to elicit qualitative information about inter-temporal relationships that allow us to infer some information about the relationships between the elicited distributions. The elicitation of qualitative inter-temporal information rather than conditional probability distributions does entail a considerable loss of information, but also makes the elicitation task much more manageable.

Our approach can be summarized as follows:
1. Decide on the time points at which to elicit probability distributions
2. Elicit probability distributions at selected time points
3. Elicit inter-temporal information
4. Construct probabilistic projections

3.1. Deciding what time points to use

The choice of how many time points probability distributions should be assessed at can of course only be answered on a case-by-case basis. It depends on a number of factors but is primarily a trade-off between obtaining an accurate representation of the entire judgemental time series and the time and effort that experts are willing to expend. Both of these, but primarily the latter, are problem-specific. Some questions that may help to set a rough guideline are (with relevant literature provided in parentheses):

1. What is the time horizon of the forecast, both in terms of time (e.g. years) and number of time-points?
2. How fast does the underlying process change? Might new trends or structural changes emerge over a period of months/years/decades? How frequently might these occur?
3. How many points (e.g. quantiles, probabilities) are to be assessed for each probability distribution? (Garthwaite et al., 2005; O'Hagan et al., 2006)?
4. How “difficult” are experts likely to find the assessment? Are they used to thinking about probabilities? Is the information likely to be “available” to them (in a cognitive sense)? (O'Hagan et al., 2006; Gigerenzer, 1996)?
5. Will they want to make rough calculations or workings during the assessment?
6. How long might it take to assess one distribution? (Garthwaite et al., 2005; Knol et al., 2009)?
7. How much time is available for this part of the assessment?

3.2. Elicitation of probability distributions at selected time points

Our protocol is largely a summary of three commonly used protocols: the Stanford/SRI protocol, Morgan and Henrion’s protocol, and the Wallsten/EPA protocol (Morgan and Henrion, 1990) and is described relatively briefly here (see e.g. O’Hagan et al., 2006). The protocol is divided into five stages. Before the interview procedure, experts are sent a set of documents containing relevant background information. The interview procedure itself comprises three stages: establishing rapport with the expert; eliciting the expert’s qualitative view of the problem, including factors influencing the outcome of interest; and eliciting the expert’s probabilistic forecasts. After the interview, the elicited judgements and statements derived from experts are sent to them to verify that they are consistent and accurately reflect the expert’s beliefs.

3.2.1. Stage 1: pre-elicitation

Prior to the interview, experts were asked to read three short documents: one summarizing the e3 model, one summarizing the available literature and points of view for the quantity to be assessed, and one summarizing the literature on heuristics and biases in probability assessment.

3.2.2. Stage 2: establishing rapport

The goal of this stage is to introduce the elicitation team to the expert and provide an overview of the reason for the elicitation and the underlying problem at hand. Following a brief introduction of the team, we explained the e3 model and the projections to be made, emphasising the uncertainty that exists around each of the key drivers to the model, and hence introduced the need for probabilistic forecasts. We then briefly noted the difficulties of long-term forecasts, emphasising that there is no “correct” answer to any of the elicitation questions, and that our main aim is to obtain judgments that reflect the expert’s expressed beliefs, in particular the extent of their uncertainty, which may be large. Finally, experts were informed that if at any stage they felt truly unable or uncomfortable making numerical probability judgments, other qualitative elicitation techniques were available, although none ultimately made use of this option. Using the pre-interview document as a basis for discussion, experts were familiarised with the dangers of subjective probability assessment, particularly overconfidence.

3.2.3. Stage 3: qualitative elicitation of factors influencing key drivers

The goal of this stage is to get the expert to think critically about the problem at hand, and to identify, in a qualitative way, the important factors that should influence their later quantitative judgments, together with some assessment of what kinds of changes are possible. To this end experts were asked two main questions. Firstly, what factors influence the key driver on which their expertise was being sought? And then, how might these factors combine in the future, in particular to cause especially low or high values in the key driver? These questions were used to generate a two-way discussion between the expert and the elicitation group, often lasting several hours, and which typically produced two distinct outcomes: firstly, a detailed qualitative description of the system relating to the key driver (the basic macro-economic model in the case of GDP growth, or the coal production and processing systems in the case of the coal price, for example); secondly, a form of “best-case” and “worst-case” scenario consisting of a qualitative storyline that might result in a particularly high or low value for the key driver. These views, which may well be at least partially constructed or modified as part of the process, form the justification for the later quantitative elicitation and as an audit trail for interested parties in the future. Many qualitative forecasting texts emphasise the need to avoid a status quo bias by encouraging broad thinking and the consideration of alternate viewpoints (e.g. Schoemaker, 1995; Chermack, 2004). We repeatedly prompted experts to consider how outcomes other than the ones that they had already specified might arise, using questions such as “what might cause a sustained GDP growth rate of less than 1%?” if that had not yet been offered as a potential outcome.

3.2.4. Stage 4: quantitative elicitation

This stage contains the formal assessment of probabilistic information. To keep the task manageable, we did not attempt to assess detailed probability distributions but rather assessed three-point (minimum, mode, maximum) distributions, modifying these where additional information (on intermediate quantiles, for example) was offered. In order to avoid anchoring on central values, we began by asking the experts for extreme lower or upper values, where additional information (on intermediate quantiles, for example) was offered. In order to avoid anchoring on central values, we began by asking the experts for extreme lower or upper values, although some experts insisted on starting with central values, and these requests were accommodated. All experts were more comfortable providing information first for 2015, then for 2030, and finally for 2050. Although this might lead later estimates to be biased towards 2015 values — which would, under normal conditions, be associated with overly narrow confidence intervals — again, we felt that it would be counter-productive to force anything else. In all cases, we attempted to combat overconfidence and overly narrow confidence intervals by asking the expert to think of scenarios that would result in values more extreme than the extreme values just given. Once ranges of extreme values had been
given, we asked for a modal (most likely) value.

3.2.5. Stage 5: post-elicitation verification

The aim of this stage is to present the expert with his or her elicited qualitative and quantitative judgements, to check that this reflects their views accurately, and to revise judgments as necessary. Ideally, this step would be conducted as part of the interview procedure but, because our interviews were already lengthy, we elected to send feedback to experts by email after the interview had been concluded. Feedback included a summary of their qualitative descriptions of the system and major influences of the key drivers and plots of the triangular probability density function obtained from their quantitative assessments.

3.3. Eliciting inter-temporal information

The results of the previous elicitation process are probability distributions defined over values of key inputs at specific time points, or mean values of the drivers over specified time periods. In order to construct a single projection from these distributions we need to (a) draw an observation from each of the probability distributions, subject to some correlation structure, which then defines the value that the projection takes on at each of the selected time periods; (b) generate values for all time periods other than the selected ones, using some form of interpolation procedure. Each of these steps requires that inter-temporal information be elicited from the expert.

3.3.1. Temporal relationships between elicited distributions

The goal here is to specify any desired relationships between random variables at the $P$ selected time points. Some of this information can be expressed as correlations, although some common temporal relationships are not easily expressed as a single correlation coefficient. For example, constraints acting on an environment may make consecutive extreme observations (either very high or very low) unlikely or impossible. In our elicitation of GDP growth rates experts indicated that economic and political constraints impose that extremely high or low values should be followed by periods of average (or at least, much less extreme) values. These constraints are causal in nature and thus should not be confused with statistical “regression to the mean” (Barnett et al., 2005), which although having the same effect (an extreme observation will tend to be followed by a less-extreme one) is a purely statistical phenomenon, without any causal interpretation. By generating values at key time points subject to some correlation structure, statistical regression to the mean will automatically be generated where appropriate (see Algorithm 1 in Appendix A). In contrast, the phenomenon we refer to here is an additional causal constraint on the types of patterns that may emerge over time. To avoid confusion we refer to this kind of inter-temporal information as indicating “moderating constraints” on the system.

In our approach inter-temporal information is elicited qualitatively through a discussion centred on a number of key questions. These are essentially qualitative interpretations of questions typically used to elicit the main features of conditional quantiles, as discussed for example in O’Hagan et al. (2006).

1. Will high (low) values at one time-point tend to be followed by high (low) values at the next time-point?
2. Is it plausible for very high (low) values at one time-point to be followed by very low (high) values at the next time-point?
3. Will high (low) values at one time-point tend to be followed by high (low) values two periods into the future?
4. How strong are these relationships (strong, moderate, weak)?
5. Are sustained periods of very high or low values plausible, or are there “moderating constraints” on the system? How long might sustained periods of extreme values last?

While difficult to measure objectively, our experience from the applications reported below is that answers to the questions above are sufficient to broadly characterize inter-temporal relationships but do not provide exact parameter values for functional forms. For example, in our applications it was extremely clear whether sustained extreme values were impossible, possible but unlikely, or likely, and why this was the case; but it was not possible to determine inter-temporal correlations precisely, experts being willing to express these only in qualitative terms (e.g. “moderate”). In that case, correlations need to be assumed and generated as part of the Monte Carlo simulation (e.g. uniformly between 0.3 and 0.5 for “moderate” correlations). As with any parameters, the robustness of conclusions to any assumed correlations can and should always be tested as part of a post hoc sensitivity analysis (e.g. Kleijnen, 2005). We discuss our experience using these questions in more detail in later sections.

3.3.2. Interpolating between selected time points

Since we elicit probability distributions at only a subset of all time periods, we require some way of inferring values at other time points. Linear interpolation between elicited values is the simplest approach and may often be adequate, given the magnitude of other uncertainties, but also suffers from two related drawbacks: it almost certainly under represents the period-to-period variability in the underlying time series, and the interpolated values between two time-points are constrained to lie between the values at those time-points. We therefore also gather, as part of the elicitation process, information that allows us to make an informed decision about whether to interpolate linearly or using a more complex approach, such as the random walk algorithm described in Section 6 and Appendix A. Standard interpolation techniques (e.g. splines, see Silverman, 1985) may also be used, but as there will usually be only a small number of elicited time points, these should be used with caution and will in most cases require additional qualitative information. Where linear interpolation is deemed inappropriate, we can also use the discussion to set some broad ranges on parameters of whatever non-linear interpolation process is used. We elicited this information based around the following key questions:

1. How much annual variability might there be? What is the maximum possible change in the process from one period to the next?
2. Should interpolated values between two time-points be constrained to lie between the values at those time-points?

3.4. Constructing probabilistic projections from the elicited information

Once information has been elicited from each expert, a key issue becomes how to integrate these assessments into a single aggregated assessment that in some sense represents the combined opinion of the experts. There is a large literature on the aggregation of expert opinion when these are represented as probability distributions, reviewed in e.g. French (2011), who distinguishes between two broad types of aggregation: behavioural aggregation, which puts the experts together and attempts to find a consensus between them through discussion, and mathematical aggregation, which uses some formal aggregation rule to combine probability distributions elicited independently from each expert.

Within the area of mathematical aggregation, two approaches are popular. Opinion pooling methods weigh the opinion of each
expert either linearly or non-linearly e.g. \( F_{all} = \sum_{e=1}^{E} w_e F_e \) or \( F_{all} = \prod_{e=1}^{E} F_{e}^{w_e} \), where \( F_e \) and \( w_e \) is the probability distribution and weight assigned to expert \( e = (1,2,\ldots,E) \). Research here focuses on the functional form of the aggregation and the determination of weights for the experts. Bayesian aggregation approaches model expert judgments as data, which can be aggregated into a likelihood function and combined with prior beliefs to form a posterior probability distribution over the event of interest.

The approach that we use is a linear opinion pool, although the presence of inter-temporal information relating the probability distributions elicited from each expert requires some care in implementation. Note that the linear opinion pool is a mixture distribution, with the weights determining the mixture parameters. Thus one can either sample \( N \) values directly from the aggregated distribution \( F_0 \) or collect together \( w_e N \) values sampled from each of the expert distributions \( F_e \). The same is true of sampling projections rather than values, except that inter-temporal information makes the calculation of \( F_{all} \) much more difficult. Aggregating over correlations, for example, is inappropriate. We therefore obtain the same result i.e. a linear opinion pool at each intermediate time-point, in a computationally straightforward way by simulating \( w_e N \) projections from the information provided by each expert. The result is a linear opinion pool for projections.

We generate probabilistic projections for each expert by first simulating randomly from the elicited probability distributions, taking into account any desired inter-temporal relationships, and then by simulating values between elicited time points, again taking into account elicited information. Different simulation approaches may be required for the first step depending on the nature of the inter-temporal information. In Appendix A we describe two algorithms that can be used when inter-temporal information is expressed as a matrix of correlations (Algorithm 1) or in the form of statements to the effect that sustained periods of high or low (Algorithm 2). Interpolation between elicited time points can be done using linear interpolation, or more sophisticated methods. Algorithm 3 (see Appendix A) provides a random walk algorithm for cases in which probability distributions are defined over values of key inputs at specific time points, while a “Gaussian” approach for cases in which probability distributions are defined over mean values is described in Section 5.

4. Application to e3 forecasting models

The context of our application is the use of SATIM, the South African implementation of The Integrated MARKAL-EFOM System (TIMES), to generate probabilistic forecasts of GHG emissions in South Africa. Although our elicitation process is entirely independent of SATIM some background may be useful, as the results of the elicitation are directly used as inputs to SATIM. Ultimately, the result of the elicitation process is, for each input variable, an ensemble of (typically many) input projections. A single projection is then drawn from the ensemble of input projections for each input variable, and passed to the SATIM model. The SATIM model returns a single output projection for any output variable of interest (since the model is deterministic). This process — sampling input projections, passing these to the SATIM model, and obtaining a single output projection (per variable of interest) — is then repeated as per usual Monte Carlo simulation to generate, for each output variable, an ensemble of (again, typically many) output projections. The sampling of projections can be done either at random or taking into account correlations between input variables where these are available.

SATIM, summarized in Fig. 1, is an inter-temporal bottom-up partial equilibrium linear optimization model of South Africa’s energy sector (Energy Research Centre, 2013). Energy demand in SATIM is modelled as demand for “useful” energy services such as cooking, lighting, heating and cooling, pumping, etc, modelled separately in five demand sectors (industry, agriculture, residential, commercial and transport). Services supplied to each of the five sectors are driven by technologies that require energy, with the quantity of energy required depending on the efficiencies of the technologies used.

SATIM uses mixed integer linear programming to select the mix of supply-side technologies that meets this demand for final energy at least cost. Final demand for energy is calculated endogenously in SATIM based on the optimal (i.e. least cost) mix of demand-side technologies that satisfies the exogenously given demand for useful energy services. Thus the optimization returns the supply-and-demand technology mix (e.g. capacity, new investment, production and consumption) resulting in the minimum discounted system cost for meeting energy demand subject to all other constraints (e.g. future energy demand, required reserve-margins, limits on resource reserves). This allows for interaction between demand and supply sectors, and explicitly models process changes, fuel switching, and efficiency gains achieved by improvements to technology.

System parameters encode assumptions for each sector, broadly covering: (a) the structure of the sector and its energy services as it impacts on the demand for energy; (b) the establishment of base year demand for energy in the sector; (c) technical and cost parameters of the technologies available to satisfy the demand for energy services currently and in the future; (d) the projection of future demand for energy services. These are described in detail in Energy Research Centre (2013) and are regularly updated for consistency with national resource plans (e.g. Department of Energy, 2013). Emissions forecasts are directly obtained from the optimal supply-and-demand technology mix returned by SATIM.

SATIM is a fairly typical example of a bottom-up e3 model, being constructed from technical process models that explicitly represent different energy technologies and the physical relationships between energy sources and outputs. Such e3 models have been extensively used to address policy issues around mitigation and planning in response to potential climate changes (see e.g. Aman et al., 2011; Azar, Lindgren and Andersson, 2003; Hedenius et al., 2013; O’Neill, Riahi, and Keppo I, 2010; Richels and Blanford, 2008; Rozenberg et al., 2010; Sassi et al., 2010). Current and earlier versions of the SATIM model have been used for policy research in Winkler et al. (2009), Winkler et al. (2011), Altieri et al. (2016) and Arndt et al. (2016).

The expert elicitations reported in the following two sections are based on a baseline scenario with no climate policy measures for South Africa, without necessarily imposing business as usual globally. That is, we include the possibility that global steps are taken to mitigate climate change but that, for whatever reasons, South Africa maintains a “business as usual” trajectory, not implementing its “Copenhagen pledge” to a 34% deviation below BAU by 2020 and 42% by 2025.

5. Expert elicitation of GDP growth trajectories

The elicitation of GDP growth trajectories is illustrative of a fairly “easy” elicitation process. We independently conducted elicitation interviews with two experts on the subject of GDP growth, both senior academic macroeconomists at the University of Cape Town. The elicitation team consisted of two analysts, one with a background in decision analysis and one working in energy modelling. Interviews took place in May and July 2014.
5.1. Summary of qualitative discussions

Discussion with each expert lasted 1.5—2 h, split roughly equally between two topics: factors influencing GDP growth in South Africa, and future trends and scenarios. For this particular input variable, discussion around influential factors was remarkably consistent between experts, both of whom began reasoning from the standard macroeconomic model given by the Cobb-Douglas production function $\text{GDPR} = (\text{TFP})K^aL^b$, where GDPR is real GDP; $K$, $L$, and TFP are capital, labour, and total factor productivity, and $a$ and $b$ are parameters to be estimated. A variety of factors influence each of these inputs in turn, and this formed the majority of the subsequent discussion. Although each expert thought slightly differently about the interrelationships between these influences, a number of common features were observed: the relatively low magnitude of savings in South Africa, the crucial role of investment in driving any future GDP growth, and political and socio-economic constraints on increasing investment in a country like South Africa.

Both experts independently highlighted that GDP growth rates should be moderately positively correlated over time, but that strong “moderating constraints” (see Section 3.4) mean that extremely high or low GDP growth values would very likely not be sustainable for longer than one time period, but would rather be followed by a period of average (or at least, much less extreme) growth. Large GDP growth rates driven primarily by improving skills in the South African labour force are naturally self-regulating, slowing as a greater proportion of the workforce becomes highly skilled. Very low GDP growth rates were felt to be politically unsustainable, leading eventually to more radical changes in macro-economic policies or to a change in government. Thus, there would appear to be strong causal factors limiting sustained GDP growth from above and below.

This led quite naturally into the experts sketching some possible “high”, “low” and “in-between” scenarios for GDP growth in South Africa over the three periods 2014—2020; 2020—2035; 2035—2050. These were assessed as qualitative, internally consistent stories involving, for example, changes in political policies, trading relationships, sectoral contributions, etc. Again the two experts showed a large degree of agreement in their qualitative scenarios, which had in common a perception of little or no change in the near future (2014—2020), limited prospects for the future but a hard floor beyond which GDP growth was felt to be unlikely to fall, and “moderating constraints” over the long-term.

5.2. Quantitative forecasts

Both experts asked to think about GDP growth in terms of a mean growth rate (in %) over three intervals (2014—2020, 2020—2035, 2035—2050), rather than the annual growth rate in 2020, 2035, 2050. The elicited probability distributions thus covered possible values in the mean growth rate over these three periods. In contrast to the qualitative discussion experts constructed their quantitative forecasts in quite different ways. It is illustrative to briefly consider these differences.

One expert provided direct answers to elicitation questions about effective maximum, minimum, and modal growth rates at each time period under the constructed scenarios. The other expert made use of a back-of-envelope calculation of the incremental capital-output ratio, $\text{ICOR} = (I/\Delta \text{GDPR})/\Delta \text{GDPR}$, where $I$ is investment, so $I/\Delta \text{GDPR}$ is the investment rate. Reasoning that South Africa’s investment rate is currently around 20%, giving an incremental capital-output ratio of roughly $\text{ICOR} = 0.2/0.035 = 5.7$, a desired
GDP growth rate of 6% would need the investment rate to be in the region of $5.7 \times 0.06 = 34\%$ i.e. almost doubled. This was felt to be a highly optimistic prospect, and thus a mean growth rate of 6% was given as an upper bound. Similar calculations were used to assist the elicitation process. Fig. 2 shows summaries of the trapezoidal probability distributions elicited from the two experts and a histogram showing an aggregate linear opinion pool (“Agg”), constructed by averaging the expert’s distributions at each time point (standardized to have the same height for each time period). Note that the aggregate distribution is shown for illustrative purposes only, and is not used to create projections – doing so would ignore differences in the inter-temporal information provided by each expert. Rather, we simulate $N/2$ projections from the information provided by each expert, where $N$ is the desired number of projections, the result being a linear opinion pool for projections (see Section 3.4 for details).

5.3 Post-processing

Simulated projections were constructed by first simulating from the experts’ elicited distributions and inter-temporal information using Algorithm 2 with $p = 0.4$ (see Appendix A for details). These were converted into annual time series by simulating, for each period of changes in GDP growth have been as large as 6% and we thus set $t$-simulations in period boundaries. We therefore reordered the simulated values changing within-period moments. Annual time series generated in quantiles, at each time point. Note that these are not true probability distributions elicited from the two experts and a historical absolute year-to-year changes in GDP growth have been as large as 6% and we thus set $\sigma_p = 0.01$. As means can change abruptly between periods, simulated values generated in this way can show sharp discontinuities at period boundaries. We therefore reordered the simulated values within each period, by selecting a value $x_{tp}$ with probability inversely proportional to the difference between $x_{tp}$ and $x_{(t-1)p}$. This scheme reduces the size of differences between the last observations in period $p$ and the first observations in period $p + 1$, without changing within-period moments. Annual time series generated in this way were aggregated using an equal-weight linear opinion pool. Sample trajectories generated by Monte Carlo simulation are shown in Fig. 3. The green line shows the median value, the red lines the 2.5% and 97.5% quantiles and the blue lines the 10 and 90% quantiles, at each time point. Note that these are not true projections, as the summaries from which they are constructed (e.g. medians) will in general drawn from different projections at each time point. As a result, they tend to be much smoother than the original projections from which they are drawn.

The projections in Fig. 3 show that annual GDP growth rates may vary between approximately 0% and 6% to 2020, and between 0% and 8% thereafter. Before 2020 very low or very high growth rates are immediately followed by much less extreme values. This is essential to keep the 2015–2020 mean within the narrow bound of 2.5–3.5% specified by the experts, and results in the “sawtooth” pattern clearly visible in Fig. 3. Beyond 2020 various scenarios are possible: sustained periods of very high or very low growth are possible, although again these are eventually followed by counterbalancing forces that keep mean growth within desired bounds. More likely though, are more moderate projections that consistently keep annual growth between 2 and 4.5%, as shown by the dense cluster of projections occurring roughly between these bounds.

6. Expert elicitation of coal prices

The elicitation of coal price trajectories is illustrative of a “difficult” elicitation process. We independently conducted elicitation interviews with four experts from a range of backgrounds. One had extensive experience at a large mining company; another had both public sector (electricity generation) and private sector (consulting to various mining operations) experience; a third was a geologist, also with public and private sector experience but primarily consulting on reserves and resources; and the fourth was an energy researcher who had recently completed a study of the coal industry. The elicitation team consisted of three analysts, one with a background in decision analysis, one working in energy modeling, and one researching institutional arrangements in the coal industry. Interviews took place between April and August 2014.

6.1. Summary of qualitative discussions

Discussions lasted between 1.5 and 3 h and covered three broad areas: the history of the coal industry in South Africa, which exerts a strong influence over the current context; factors influencing the price of coal in South Africa; and future trends and scenarios.

Experts tended to be in general agreement about broad themes regarding the past and present of the coal industry in South Africa. The future of the coal industry, and hence of coal prices, was characterized as one of massive systemic uncertainty. All experts saw the displacement of coal by other energy sources as a distinct possibility, as rising production costs (or falling production costs of competing sources) could make coal use economically unviable, even under assumed “business as usual” policies. Here we had to emphasise that the decision about coal’s economic viability would be made endogenously within SATIM, which chooses fuels on a least-cost basis. Thus other sources can displace coal in our model if economics dictates that this should happen – experts need not explicitly consider viability, although they intuitively avoided more extreme scenarios in which coal was definitely not economically viable. Further discussion thus largely assumed that coal could potentially be economically viable under a scenario, and focused on producing the range of costs that were possible under each of the constructed scenarios.

Experts took both a “top down” and “bottom up” view of the coal system, but typically preferred to think primarily in terms of one. The “top down” view emphasized broad economic, social and political trends. This view emphasized the complex inter-relationship between the local market for low-quality coal used for electricity production, and the export market, historically for higher-quality coal but increasingly also for low-quality coal. The local market has been formed around long-term (often lasting 30 years or more) contracts typically negotiated between large mining houses and a state-owned electricity producer on “low risk, low return” lines, allowing mining companies to make the large capital investments needed to start and operate a mine. Almost all of these long-term contracts end within the next 5–15 years and will need renegotiating.

Complicating this process, many of South Africa’s coal-fired power stations are near the end of their lifespans and will need

---

Fig. 2. Elicited triangular and trapezoidal probability distributions for mean annual GDP growth rate (2 experts). Where one symbol is shown, this represents the mode of a triangular distribution. Where two symbols are shown, this means the expert expressed the mode in interval form (“between L and U”).
major refurbishments or decommissioning, while many mines paired to these stations do not have the capacity to supply coal at a reasonable cost for another contract cycle. Mining companies are unlikely to undertake the risk of establishing or upgrading a mine unless government first ensures a market for it, requiring massive capital outlay for new power stations and associated infrastructure. This is politically and economically difficult for government. At the same time, the export market has come to increasingly compete for coal traditionally destined for the local market, either through the “washing” of coal to higher grades, or demand from India and China, which now also use low-grade coal for electricity generation. Mining companies are therefore likely to demand rates of return well in excess of historical norms. At present, while this longer-term process plays out, increasing amounts of coal are transacted using short-term contracts. These tend to be small volume, expensive, and difficult to manage logistically.

A “bottom up” view begins with more specific input factors influencing the price of coal: the costs of mining it, the necessary return on capital; logistics (mainly transportation costs by road or rail); labour costs; energy inputs (in the form of diesel and electricity); capital expenditures and the associated required rates of return on capital; environmental and social costs (acids mine drainage, royalties/licensing, carbon tax); the growth of the low-grade export market; and assorted “other” costs (water costs, engineering costs, replacement capital costs, employee housing costs, and equipment costs). Each of these factors is a complex subsystem subject to major uncertainties regarding its future, but many of these can be analysed in a fairly technical manner. Mining costs, for example, are a function of the geology, location, and extraction process. Transportation costs are a function of the geographic proximity of mines and power stations, the efficiency to which they are paired, and the relative importance of the export market.

The qualitative discussions were difficult and time-consuming, and progress towards scenario building was slow. Scenarios, when constructed, tended to reflect the simultaneous presence or absence of a few key events or input variables. All experts strongly expected the cost of coal to rise substantially, and were pessimistic about the fortunes of the industry. No major changes were expected in the near-term (to 2020). To 2035 the primary uncertainty is how high return on capital and infrastructure (including transport) costs will rise once contracts are renegotiated. Labour costs, water costs, environmental legislation, and royalties contribute in a lesser way. In the longer term, the primary uncertainty is around the viability of coal in the Waterberg, an area of South Africa containing a vast amount of mostly low-quality coal where uncertainties around operating and transportation costs are considerable. Here experts created competing scenarios in which new power stations were built in the Waterberg to minimize costs, or vast transport networks were created to ship Waterberg coal to the Central Basin area where most power stations are currently located and/or harbours for export. All uncertainties relevant to the 2035 time horizon remain relevant, particularly as mines that negotiated their contracts in 2020–2030 will again be coming to the end of their contracts by 2050.

6.2. Quantitative forecasts

The quantity elicited was the average price of coal that coal-fired power stations (which use low- or medium-grade coal in the 18–22 MJ range) would need to pay at the plant gate, in the reference years 2020, 2035, and 2050. Experts expressed these prices using different monetary units (2010 Rand/ton, 2012 Rand/ton, or 2014 Rand/ton). These were adjusted to a common scale (2012 Rand/ton) in post-processing. All experts had difficulty assessing future coal prices and thought there was perhaps a feeling that, even within a fairly detailed scenario, too much had been left unspecified. This must be interpreted as a failing of the scenario construction process although, with limited time available to interview each expert, it is difficult to see what we could have done much differently.

Although quite similar in the construction of influences and scenarios, each expert thought quite differently when quantitative judgements were elicited. One expert’s thoughts (E4) could be assessed in the traditional way, as they had provided direct estimates of possible coal prices, while the other three preferred to use some form of more-or-less rough calculation to guide their judgements. Thus a second expert (E3) selected a set of key drivers relevant to each time period, and for each sketched the types of conditions that would lead to high, moderate, and low coal prices. Thinking about the combinations of conditions that lead to low (moderate, high) coal prices, the expert could give an estimate of the price under these conditions. The third expert preferred to think in terms of supply curves (E2), relating the available quantity and expected mining cost of known deposits in South Africa. These had been partially constructed for the purposes of another exercise, but were in a deterministic format. By thinking about the uncertainty around quantity and cost, we were able to assess distributional estimates for the current project. The fourth expert (E1) decomposed the cost of mining into logistics, labour, energy inputs, return on capital, acid mine drainage, royalties, carbon taxes, and other costs. For each time point, a plausible range of costs was given for each input, with notes describing what might cause a particularly high or low value. These were combined to give aggregate cost ranges.
that to the extent that coal continues to be used, it will be mined from new mines in the Waterberg area. Thus coal prices paid by Central Basin power producers, which include substantial transportation costs from the Waterberg mines, rise sharply between 2020 and 2030 as Central Basin supplies run out (Fig. 5a). The model predicts the construction of new power plants in the Waterberg, which are able to avoid transport costs and thus pay a substantially lower price for coal (Fig. 5b). The average price weights Central Basin and Waterberg prices by the number of power plants in each region: the decrease in the median coal price after 2030 occurs as most Central Basin power plants are decommissioned and Waterberg plants become relatively more prevalent (Fig. 5c).

7. Discussion

Three key features made eliciting future coal prices far more difficult than eliciting future growth rates of GDP: Firstly, although the specifics of GDP growth are technical, the broad economic influences are well studied and understood, and easily conveyed. The standard macroeconomic model \( GDP = (TFP)kL \) provides both an entry point for more detailed discussion and a basic skeleton from which other model components can be hung. The coal industry in South Africa is very different. It is a large, highly complex system that can be viewed in a number of different ways (supply/demand; top-down/bottom-up). That is, whereas for GDP growth there is a single well-defined and largely agreed-upon view of the “system” as a whole, in the case of coal we needed to construct this view as part of the elicitation process.

A second, related point is that the boundaries around the GDP system are generally well defined, as indicated by the standard model. Most of the qualitative elicitation time was given to discussing determinants of labour, capital, and total factor productivity inputs, and the extent to which these might change over time. The problem in the case of coal prices is that these depend on both the production cost of coal and on the demand for coal. Both of these are highly complex and depend on a number of other factors that, in the case is final coal demand, are endogenously determined within SATIM and thus cannot be easily integrated into the elicitation process. As the boundaries around the problem grow, it also becomes increasingly difficult to find experts who are able to assess the whole system. Our “solution” in the case of coal was to reconstruct, on the basis of elicited information, the coal supply curve i.e. the costs, including return on investment, of producing various quantities of coal, but to generate coal prices as an endogenous output of the SATIM model. Thus in our application attempts to directly elicit future coal prices led us to revise the SATIM model to accept uncertain coal supply curves as input.

A third distinguishing factor is the relative stability of the two systems. As is clear from the elicitations relatively little change in GDP growth is expected, particularly in the short term but even, while recognizing the inherent uncertainties, far into the future. Some deeply entrenched features constrain GDP growth in South Africa from above and below, and these appear unlikely to change altogether. Of course these might change, but GDP growth in South Africa appears to be viewed, at the present time, as a relatively stable system. All experts, in contrast, characterized the coal industry, as being in a state of flux, and subject to large and unresolved uncertainties. These include uncertainties around the renegotiation of long-term contracts discussed in Section 5 as well as other uncertainties. The South African government has made a number of recent public statements indicating plans to develop shale gas resources and build nuclear power stations. While none of these had reached an advanced stage of planning at the time of writing (June 2016), these pronouncements further increased uncertainty around elicitions of coal prices.

Fig. 4. Elicited triangular probability distributions for domestic coal prices (4 experts).
Despite their difficulty the qualitative discussions were crucial in allowing the experts to structure their thoughts and opinions clearly, and in recording many of the assumptions behind subsequent quantitative estimates. The quantitative estimates are of course the main outcomes of the elicitation process. For both GDP growth and coal prices, experts were largely in agreement about the key properties of the forecasted process — relative stability and moderating constraints in the case of GDP growth, steep increases in the case of coal prices, and increasing levels of uncertainty over the medium term in both series. Large difference in the ranges of coal prices elicited from experts at 2050 (from R280/t (expert E1) to R714/t (expert E4)), largely reflect beliefs about the price levels at which coal remains viable i.e. can be sustainably produced and sold, relative to other energy sources. Resolving this question proved extremely complex and, arguably, beyond what could be elicited in a reasonable amount of time, even from experts. For this reason we reconstructed approximate coal supply curves from the elicited information and computed coal prices endogenously using our e3 model. With hindsight, it would have been preferable to limit the elicitation process to the assessment of coal supply curves. This was not, however, obvious before or even during the elicitation process.

In the final analysis, the range of possible coal prices returned by our e3 model and shown in Fig. 5 (R300/t – R900/t if one considers both the Central Basin and Waterberg regions, or a weighted average of R300/t – R650/t) should probably be considered conservative, and constrained by assumptions made by the e3 model that may not necessarily hold over long periods of time. This is of course a fundamental limitation of using an e3 model to construct long-term projections of any kind.

8. Conclusions

Explicitly representing uncertainty is generally recognised as a fundamental requirement of any long-term forecast. The current paper proposes and illustrates the use of expert elicitation to construct long-term probabilistic projections. Each projection represents a plausible future evolution of a quantity of interest. By generating, via Monte Carlo simulation, many possible trajectories, one can easily assess the distributional properties — and hence uncertainty — of the process at any point in time. Our context in using this elicitation approach is e3 modelling in South Africa, but it should be clear that the methodology is broadly applicable to the elicitation of many time series with autocorrelation properties. In our application, the projections constructed from the expert elicitation were passed to e3 models to generate forecasts, also in the form of probabilistic projections, of greenhouse gas emissions under the assumptions made by the e3 model (generally, no fundamental or extreme changes to the system from climate impacts e.g. dramatic rises in sea level, etc).

Our approach simplifies the elicitation of projections by eliciting detailed probability distributions at a few points in time, and then reconstructing projections for the entire time from these distributions using inter-temporal information also gathered as part of the elicitation process, and a small number of simple interpolation algorithms. Our approach is pragmatic, in the sense that while it entails a substantial loss of detailed quantitative information (that would in principle be collected by, for example, a series of conditional probability distributions) it allows for information about complex processes to be elicited in a reasonable amount of time. In our two case studies, we were able to assess 35-year forecasts of GDP growth rates and coal prices in an interview that lasted between 3 and 5 h, depending on the expert.

With regard to further work, a number of possible avenues exist. Our results show that uncertainty increases exponentially over time. Median projections can be calculated, but are no prediction of the future. Forecasts should be regularly updated, perhaps at intervals of no more than five years. Our projections are based on the assessments obtained from only a small number of experts; increasing the number of interviewed experts is an important practical task. Although autocorrelation within each key input variable is modelled explicitly, we have also not addressed the difficult problem of assessing correlations between input variables. The difficulty in this regard is simply finding experts with sufficient knowledge to assess these correlations. Experts exist with subject areas, but the assessment of inter-variable correlations requires an extremely broad and deep knowledge, encompassing all the input variables. Elicitation methods and practical applications for these would be valuable and timeous contributions.

Acknowledgements

This research was supported by grants from the United Nations Environmental Program (UNEP) and Mitigation Action Plans and Scenarios (MAPS) International. Any findings, conclusions, recommendations, or opinions expressed in this document are those of the authors and do not necessary reflect the views of UNEP or MAPS.
Appendix A. Details of post-processing of elicited data sources

Algorithm 1: generating correlated random samples from arbitrary distributions

Suppose we wish to simulate \( N \) values from each of a set of \( P \) arbitrary distributions \( F_1, F_2, \ldots, F_P \) with the resulting simulated values having a correlation matrix \( \Sigma \). An algorithm for doing this to good approximation is:

1. Simulate \( N \) samples from a multivariate standard normal distribution with correlation matrix \( \Sigma \).
2. Convert the values generated in the previous step into probabilities by applying the univariate standard normal CDF to each of the \( P \) values, i.e. independently.
3. Simulate draws from the desired distributions by applying the inverse CDFs \( F_1^{-1}, F_2^{-1}, \ldots, F_P^{-1} \) to the probabilities generated in the previous step.

The simulated values preserve the desired correlation structure only approximately, because the transformation in step 2 preserves the rank order rather than the exact correlations. Nevertheless in general testing we found the approximation to be good, and given the inherent uncertainty in specifying the correlation in the first place, any errors introduced by the approximation are likely to be negligible.

Algorithm 2: generating random samples with “moderating constraints”

Series in which extremely high or low values should be followed by periods of average (or at least, much less extreme high or low) values pose a problem for Algorithm 1 because the desired correlation between the ranks of the remaining values has a correlation matrix \( \Sigma \). An algorithm for doing this to good approximation is:

1. Simulate \( N \) samples from a multivariate standard normal distribution with correlation matrix \( \Sigma \).
2. For each of the \( N \) values simulated from \( F_1, F_2, \ldots, F_P \) using standard methods. For each \( t \) for which moderating constraints should apply from \( t \) to \( t + 1 \):
   a. Generate three possible “moves”
      i. \( x_{et-1}^* = x_{et} + d_p - \Delta_p \)
      ii. \( x_{et-1}^* = x_{et} + \Delta_p \)
      iii. \( x_{et-1}^* = x_{et} + d_p + \Delta_p \)
   b. For each possible move, calculate the distance between the terminal point \( x_{t+1} \) and the sum of the proposed value \( x_{t+1}^* \) and the drift that is still to be added in the remaining \( \tau \) time periods
      \[ e_t^\tau = x_{t+1} - \left( x_{t+1}^* + (\tau - t) d_p \right) \]
   c. Calculate the maximum change due to random steps that is possible in the \( \tau \) time-steps that remain i.e. \( \Delta_t = (\tau - t) \Delta_p \).
   d. Select one of the proposed moves at random, where the selection probabilities are given by \( \theta_i = 1 - \max(0, (e_i^\tau / L_i - \epsilon)^2) \), where \( \epsilon \) is a small constant that prevents the selection probability going to zero where \( e_i^\tau = L_i \) exactly (i.e. where the move is still strictly permissible). Thus, where a proposed move leads to a point that is further away from the target than the maximum remaining changes that may occur, \( e_i^\tau > L_i \) and the resulting selection probability \( \theta_i \) will be zero. Moves become relatively less likely to be chosen as they approach this limit.

Note that we do not explicitly work out the final step from \( x_{t-1} \) to \( x_t = x_{t-1} \). In general, it will not be possible to reach \( x_t \) from \( x_{t-1} \) using only drift and the random change i.e. \( x_{t-1} - x_t = d_j \), but the above steps are sufficient for the final “random” change required to be smaller than \( \Delta_p \), which is sufficient for the purposes of our study.

Appendix B. Software

We developed software for the statistical computing environment R, which is freely available under the GNU General Public Licence (R Core Team, 2015). All code is contained in the package check_allpts
- Run checks on trajectories to see they match desired properties.

<table>
<thead>
<tr>
<th>Function</th>
<th>Purpose</th>
</tr>
</thead>
<tbody>
<tr>
<td>simulate_at_elic_times</td>
<td>Generate samples from elicited distributions,</td>
</tr>
<tr>
<td></td>
<td>including inter-temporal information.</td>
</tr>
<tr>
<td>check_keypts</td>
<td>Run checks on samples to see they match desired</td>
</tr>
<tr>
<td></td>
<td>distributional properties.</td>
</tr>
<tr>
<td>interpolate_betw_elic_times</td>
<td>Generate trajectories from each sample, by</td>
</tr>
<tr>
<td></td>
<td>interpolating between assessed time-points.</td>
</tr>
<tr>
<td>plot_trajectories</td>
<td>Visualize simulated trajectories.</td>
</tr>
<tr>
<td>check_allpts</td>
<td>Run checks on trajectories to see they match desired</td>
</tr>
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<td></td>
<td>properties.</td>
</tr>
</tbody>
</table>
projection. The package and an associated vignette documenting an illustrative example is available at http://dx.doi.org/10.5281/zenodo.35072. The package contains tools for generating temporal projections from elicited (or otherwise constructed) triangular or trapezoidal probability distributions. The package has two main goals: (1) generating samples at each assessed time-point in such a way that any inter-temporal information is included, and (2) generating interpolated values for time-points occurring between the assessed time-points.

The package requires the pre-installation of the R packages MASS, trapezoid, and ggplot2. It would usually employ the functions below, in the order given. It is assumed that at each of assessed time points, parameters of a trapezoidal probability distribution have been assessed in some way.

We illustrate the package with a simple example. Suppose that by some means it has been established that the distribution of GDP growth for 2014 is Triangular (2.4,2.5,2.6); for 2020 it is Triangular (2.5,3.3,5); for 2035 it is Triangular (15,3,6); and for 2050 it is Trapezoidal (15,3,4,6). Furthermore assume that GDP growth is moderately positively correlated between time-points, say with a correlation of 0.3. We capture this information as follows:

\[
x_{\text{min}} = c(2.4, 2.5, 1.5, 1.5) \\
x_{\text{model}} = c(2.6, 3, 3, 3) \\
x_{\text{mode2}} = c(2.6, 3, 3, 4) \\
x_{\text{max}} = c(2.8, 3.5, 6, 6) \\
correls = \text{matrix}(0.3, 4, 4); \text{diag}(correls) = 1
\]

To generate correlated samples at assessed time points we use `simulate_at_elic_times`, specifying the desired number of simulated values and the input information about. Moderating constraints, in the sense defined in Algorithm 2, can also be performed. For example, the line below specifies that extreme values at 2035 should be followed by less extreme values at 2050.

\[
x_{\text{e}} = \text{simulate_at_elic_times}(\text{nsims} = 200, \text{Sigma} = \text{correls}, x_{\text{min}}, x_{\text{model}}, x_{\text{mode2}}, x_{\text{max}}, \text{mean_rev} = c(0, 0, 1, 0))
\]

The `check_keypts` function performs a number of basic checks on the output of the previous step, returning three outputs: a correlation of 0.3. We capture this information as follows:

\[
x_{\text{e}} = \text{simulate_at_elic_times}(\text{nsims} = 200, \text{Sigma} = \text{correls}, x_{\text{min}}, x_{\text{model}}, x_{\text{mode2}}, x_{\text{max}}, \text{mean_rev} = c(0, 0, 1, 0))
\]

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\[
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\]

Interpolation requires only that we specify how many values to insert between each of the assessed time points, using the `nperiods` option. Note that the line below generates a projection with 37 values: annual values from 2014 to 2050. This is the sum of the elements in `nperiods` plus the four assessed time points, which are inserted in the appropriate place in the sequence.

\[
x_{t} = \text{interpolate_betw_elic_times}(y = x_{\text{e}}, \text{nperiods} = c(5, 14, 14), \text{method} = \text{`linear.from.points'})
\]

We can then perform a number of additional checks on the projections, which provide some summary plots from the generated projections: histograms of the values in each time period, histograms of the means of values in each time period, a line plot of a few randomly sampled projections, and a line plot of all the projections. In the line below, note that we use different values of `nperiods`, telling the function how to break up the 37 values in the projection into time periods: 7 observations (2014–2020), then 15 (2021–2035), then 15 (2036–2050).

\[
\text{check2 = check_alllpts}(x_{t}, \text{nperiods} = c(7, 15, 15))
\]

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