JOHANNESBURG OR GROUP

NEWS AND VIEWS

March, 1969

(Your news and views will be welcomed and published)

NEXT MEETING

Chairman:	Mr. D. Masterson
Speaker:	Mr. Mike Hattingh

Mr. Hattingh, who has an MSc. (Mathematical Statistics) is at present studying for the Ph.D. at Potchefstroom University.

He has completed a large number of O.R. and Statistical studies at SASOL.

Subject: A Large Industrial Problem in the Petrochemical Industry.

An optimization study was carried out on a section of the SASOL plant, by an O.R. team. A model was derived to relate as accurately as possible the income for the Arge Synthesis plant to operating variable levels. The paper will survey the problems encountered in the study and will describe the solution that was proposed.

Date: March 19th

Time: 8 p.m. - 9.30 p.m.

Venue: A smaller lecture theatre has been chosen to encourage participation:

It is: Room G101,
1st Floor,
North Wing,
Mining and Geology Block,
Witwatersrand University.

Agenda:

1. Chairman's Welcome
2. "A Large Industrial Problem in the Petrochemical Industry".
3. Questions
4. Matters of Special Interest
5. Tea and Talk

Next Meeting + 1

Speaker:	Mr. Bob Tandy
Subject:	"Management Games"
Date:	April 16th
Time:	8 p.m.

Last Meeting

The panel of four made a highly commendable attack on the problem. Their weapons were as varied as Simulation (GPSS), Quadratic Programming, the stock balancing trick (where the costs of carrying, running short, and ordering fight each other and the referee is the distribution of demand), and the reduction of variety.

Their concentration was so intense that in no case, when the red timing light came on, were they seen to bat an eyelid.

Their solutions will be published in the next newsletter.

SIG ACTIVITIES

The purpose of the Special Interest Groups is to help people to work together in depth, independently of our monthly meetings, on their special interests or problems. If you are interested in any of the activities outlined below, please contact the SIG leader. If you would like to lead a group in an activity not yet catered for, please inform any member of the committee.

Cybernetics:	Mr. Mike Roberts	838-3581
Dynamic Programming:	Mr. Tom Rozwadowski	48-1028
Econometric Models:	Mr. John Joslin	836-8321
Forecasting:	Dr. John Ryder	838-3581

The next meeting of this SIG will be at 15, Panoramic Heights, Bellevue, Johannesburg, from 5.30 p.m. to 7.30 p.m. (maaimum), on 2nd April, 1969. Dr. Ryder will discuss the use of the gamma distribution in forecasting.

All are welcome.

Simulation: Mr. Gert van der Veer 713-4201

The next talk will be given by Mr. Goslaer on "Simulation in the Bank", at 4 p.m. on the 17th March, in the offices of the Sigma Data Corporation, Braamfontein (Niven House).

All are welcome.

The next + 1 talk will be by Mr. Jack Curtis, entitled "South Africa vs MCC", on April 14th.

Statistical Quality Control: Dr. Sichel 724-8172

OR in the Construction Industry: Mr. V. Shaw 74-6011

Current Events

British Computer Society - 1 Day Seminar

"The Effects of Computers and Automation on People at Work"

Monday, March 24th, Loughborough Technical University - Chairman Lord Carron.

Central London Productivity Association - 1 Day Seminar

"Capital Investment Appraisal"

Wednesday, March 19th. The Carlton Tower, London S.W.1.

South Wales O.R. Discussion Society Conference.

"Information and Management"

April 17, 1969. University of Wales Institute of Science and Technology, Cardiff.

Linear Programming - Rex Border - BSc. Honours (Mathematics)

The main objective is to set out some broad requirements for the operations researcher and to describe by the use of a simple example how Linear Programming can be used to assist management in making better decisions in the field of Transportation-type problems.

Operations research as a science developed along with the development of high speed computers and it is interesting that its biggest successes have been achieved in the business field. The objective of any operations research project is to find ways and means of improving the profitability of a company by the use of computers, mathematical or statistical analysis techniques and a bit of practical application ability.

The most important part of such a study is to understand clearly the functions of the particular business;

then to accumulate as much appropriate data as possible of the operating of the company;

then to analyse this data statistically or using whatever tools are available for analysis;

then to draw conclusions as to the reliability of the analysis and to give management of the business a clear line of action and of the likely returns.

Operations research is meant to produce optimum answers to business problems. However, it is never possible to reach an optimum answer since, for example - in manufacturing, profits are the difference between sales revenue and production/administration costs and thus the optimum answer would be to produce and sell an infinite quantity of the product at zero cost.

Linear programming is a mathematical technique designed to produce optimum answers as a result of analysing various alternatives. As such it is a valuable tool. But of course the various alternatives are always subject to improvement and hence it is a dynamic tool able to analyse the alternatives as they become known.

In the field of transportation, products can be moved from various sources to various destinations by various means at a measurable cost. The objectives in this type of problem are to arrive at answers which indicate how much;
from where;
to where;
by what means;
at the lowest cost.

In order to illustrate the value of LP in this field I would like to consider a fairly simple problem.

THE TRANSPORTATION PROBLEM

Consider a firm that has two supply depots at Pretoria and Germiston. They have to meet a demand at Johannesburg, Benoni and Krugersdorp. There are limited resources available at the depots and the unit cost for transporting unit quantities of their product from source of supply to demand are known.

We have the supply and demand table as follows showing the unit cost from each source to demand point:

	Johannesburg 30	Benoni 130	Krugersdorp 30	
Germiston	10	7	3	110
Pretoria	8	8	5	100

What quantities should be delivered from which source to which demand so that the total cost is a minimum?

With such a simple problem it is easy to guess the best solution - or is it?

Germiston to Krugersdorp	=	30 @ R3-00	=	R90-00
Germiston to Benoni	=	80 @ R7-00	=	R560-00
Pretoria to Benoni	=	50 @ R8-00	=	R400-00
Pretoria to Johannesburg	=	30 @ R8-00	=	R240-00
				R1, 290-00

Some considerations arising out of this problem are:

1. Which source of supply would be the most economical to expand?
2. At what unit cost would it be economical to transfer surplus stock from Pretoria to Germiston say?
3. If, for various reasons the cost of storing surplus goods at Pretoria were R3-00 and the cost of storage at Germiston were R0-85, would this influence the result?
4. Where should a more economical supply depot be located?

To express the problem in a better form, consider the following table:

		VARIABLES						SURPLUS			
		GJ	GB	GK	PJ	PB	PK	SG	SP		
	Unit Cost	10	7	3	8	8	5	.85	3.00	Minimum	
	Germiston	1	1	1				1		110	supply
	Pretoria				1	1	1		1	100	
Conditions	Johannesburg	1			1					30	demand
	Benoni		1			1				130	
	Krugersdorp			1			1			30	

		Cost
In the initial solution	GK = 30	R90-00
	GB = 80	560-00
	PB = 50	400-00
	PJ = 30	240-00
As a result, the surplus at Pretoria SP = 20		60-00
		R1350-00

Is there a better solution?

Yes: a better solution would be	GK = 30	R90-00
	GB = 60	420-00
	PB = 70	560-00
	PJ = 30	240-00
	SG = 20	17-00
		R1327-00

By how much could the cost of storage at Germiston be increased before the solution will change?

If surplus at Germiston cost R2-00 per unit for storage then the answers would be the same. Similarly, the cost of storage at Pretoria would have to drop to R1-85 before the solutions would be equal.

In other words, by a careful study of the facts, we can readily reach definite decisions of what will be the course of action to follow if something changes.

Assume that Germiston and Pretoria could each have fulfilled the entire demand for the period ignoring storage costs for the moment.

The best solution would then be:

	Cost
PJ 30	240-00
GB 130	910-00
GK 30	90-00
	R1240-00

If Pretoria had unlimited quantities available, but Germiston were restricted to its original value then the best solution would have been the first one.

In other words, Germiston is a better depot to expand than Pretoria.

Mathematically, this problem could have been expresses as follows:

Let quantity X to be shipped from depot (i) to demand point (j) be expressed as Xij.

Let demands be represented by constants b(j);j=1, 2, 3.

Let available quantities be represented by a(i); i=1, 2.

Let the cost C per unit to be shipped from depot (i) to demand point (j) be expressed as Cij.

Let the surplus quantity S for depot i be represented by Si.

Then we are required to solve:

$$
\begin{array}{llllllll}
X_{11} + X_{12} + X_{13} & & & & + S_1 & & = a_1 \\
 & X_{21} + X_{22} + X_{23} & & & & + S_2 & = a_2 \\
X_{11} & + X_{21} & & & & & = b1 \\
X_{12} & + X_{22} & & & & & = b2 \\
X_{13} & + X_{23} & & & & & = b3
\end{array}
$$

Subject to the fact that all Xij $\geqslant$ 0 and we are required to minimise the cost function:

$$C_{11}\ X_{11} + C_{12}\ X_{12} + C_{13}\ X_{13} + C_{21}\ X_{21} + C_{22}\ X_{22} + C_{23}\ X_{23} + C_1\ S_1 + C_2\ S_2 .$$

Which is the general form of a linear programming problem.

Characteristics of this type of problem are that there are more variables than equations, there is an objective function to be minimised, no variable may be negative; all relationships are linear.

In the problem under discussion, we thus have:

Minimise $10X_{11} + 7X_{12} + 3X_{12} + 8X_{21} + 8X_{22} + 5X_{23} + 0.85S_1 + 3S_2$.

Subject to:

$$
\begin{array}{ll}
X_{11} + X_{12} + X_{13} + S_1 & = 110 \\
X_{21} + X_{22} + X_{23} + S_2 & = 100 \\
X_{11} + X_{21} & = 30 \\
X_{12} + X_{22} & = 130 \\
X_{13} + X_{23} & = 30
\end{array}
$$

With the additional requirement Xij $\geqslant$ 0., Si $\geqslant$ 0.

Consider the problem facing the competitor who wishes to break into this field -

He wishes to know what is the maximum rate he can charge for deliveries to each of the demand and/or supply points in order to make the most profit for himself as well as being competitive with the existing supplier.

From our previous problem we can now introduce charge variables Wi for each of the equations and set out a new problem as follows:

Find values for Wi that satisfy the following equations:

$$
\begin{array}{llllll}
W_1 & & + W_3 & & & \leqslant 10 \\
W_1 & & & + W_4 & & \leqslant 7 \\
W_1 & & & & + W_5 & \leqslant 3 \\
 & W_2 & + W_3 & & & \leqslant 8 \\
 & W_2 & & + W_4 & & \leqslant 8 \\
 & W_2 & & & + W_5 & \leqslant 5 \\
W_1 & & & & & \leqslant 0.85 \\
 & W_2 & & & & \leqslant 3.00
\end{array}
$$

and maximise the objective function:

$$110W_1 + 100W_2 + 30W_3 + 130W_4 + 30W_5 .$$

After careful thought, a bit of arithmetic and much head-scratching we find that:

W_1	=	0.85	x	110	=	R93-50
W_2	=	1.85	x	100	=	185-00
W_3	=	6.15	x	30	=	184-50
W_4	=	6.15	x	130	=	799-50
W_5	=	2.15	x	30	=	64-50
						R1327-00

Which is as high as the competitor can go in order to be no more expensive than the existing supplier.

Mathematics uses the name PRIMAL for the original general form of the Linear programming problem and the switched-over version is called the DUAL.

The Dual is formed by changing rows of the Primal to columns, making cost co-efficients of the primal the maximum limits for the Dual, making requirements of the Primal become the profit co-efficients for the Dual, and maximising the profit instead of minimising the costs.

A characteristic of the Dual problem is that there are many more equations than variables and hence it is a simpler problem to solve.

Mathematicians take great pains to prove that if a best solution exists for the Primal, then a best solution exists for the Dual, and vice-versa. This has resulted in quicker solutions to large problems and also has meant that it is possible to provide management with better decision-making information as a result of applying Linear programming to business problems.

TRANSPORTATION TECHNIQUES

Because of the nature of the numbers used in setting up transportation problems i.e. - all co-efficients of the matrix are unity, there are special techniques available for solving them. Their advantage lies in the fact that the matrix need only have as many rows as there are sources, and as many columns as there are destinations.

Remember that in LP, rows $= S + D$
columns $= S \times D$

Their disadvantage lies in that there is no economical information provided. This is the biggest point in favour of using LP.

Practical Aspects

Practical transportation problems are very large indeed and fairly laborious to solve. In order to satisfy the OR criterion for finding better solutions to problems it is often possible to break problems down into smaller ones and achieve good solutions if not quite optimum.

In practice too, you often have to handle more than two related variables :

For example, Truck sizes, sources of supply and demands to be satisfied. This means additional rows and columns and hence longer solution time.

Conclusion

Linear programming is a mathematical tool which is necessary to the OR worker.

The Duality aspect of LP provides valuable management information.

Most modern computers are equipped to handle LP problems and are reletively easy to use - but - get to know the problem well before attempting to solve it.

National Co-ordinating Committee

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Mr. Pirow	836-1166
Mr. Joslin	836-8321
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Mr. G. van der Veer		713-4201
Mr. D. Hawkings		724-1311

Address

Johannesburg Operations Research Group,
P. O. Box 3214,
JOHANNESBURG.

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